Names: _____

Rotate, Translate, and Scale

Answer the following questions by experimenting with Transformula (https://transformula.twodee.org/).

1. What is the inner radius (the radius of the hole) of the torus?

2. What is the outer radius of the torus?

3. What is the radius of the sphere?

4. What is the edge length of the cube?

5. Suppose you have the following rotation matrix:

$$\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   (a) What type of rotation is represented by this matrix?

   (b) By inspecting only the matrix, how can you distinguish it from the other types of rotations?

6. Using two matrices, transform the cone so that's tip is at the origin and it fans out to the right. Write down the matrix multiplication and its product.

7. You have the following gauntlet of transformations:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

You decide you want to put the model back at the origin. Instead of changing the existing gauntlet, you apply a new transformation after the existing transformation is applied.

(a) Where does the new matrix go in this product?

(b) What kind of transformation should you use?

(c) What is its matrix?

(d) Our eyes are easily fooled. How can you be sure that the model is at the origin?

8. Render the box and try transforming it with only rotation, scaling, and translation.

(a) Can you transform the box so that its faces are no longer perpendicular? If so, write down the transformation gauntlet.

(b) Can you transform the box so that its parallel faces are no longer parallel? If so, write down the transformation gauntlet.

(c) What does it mean for a transformation to be **affine**?

9. You want to produce this transformation matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & -0.96 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) What two transformation types compose to produce this matrix?

(b) Using appropriate matrices for the two types, write a multiplication that produces this transformation.

(c) Flip the order of the two types. Write the other multiplication that produces the exact same transformation.

(d) Which ordering do you find easier to tweak? Why?

10. These two transformations appear to have the same effect on the torus:

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 1.4 \\ 0 & 1 & 0 & 1.2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a)

$$\begin{bmatrix} 1 & 0 & 0 & -2.8 \\ 0 & 1 & 0 & -2.4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 1.4 \\ 0 & 1 & 0 & 1.2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

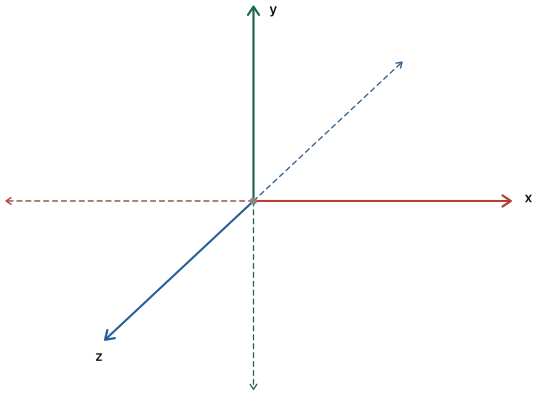
(b)

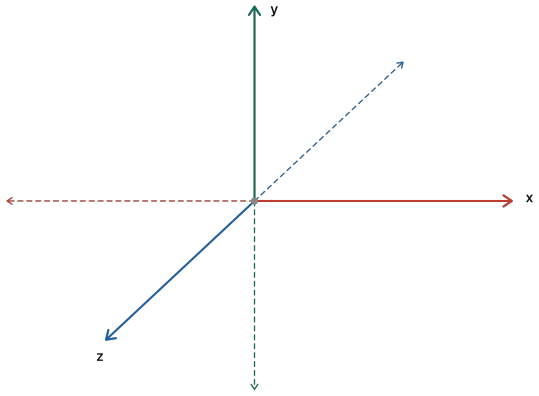
Are they the same? Why or why not?

11. You've got the following transformation:

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

You decide you to also rotate around the y-axis by 90 degrees. On the left axes, draw the transformed model if you rotate first. On the right, if you rotate second.





12. Use the custom transformation type to assemble this matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) Write an equation to compute x'—the transformed x—as a dot product.

(b) Rewrite the equation with the dot product expanded and simplified.

(c) Describe in words the effect of this transformation.