Suppose we have two arbitrary matrices A and B and we want to find $(AB)^{-1}$. In general, inverses are tedious to calculate. However, perhaps A and B separately have inverses that are simple to find. If we could find a way to express $(AB)^{-1}$ in terms of A^{-1} and B^{-1}, that'd make our lives easier. Good news. We can. First, let's remind ourselves of the definition of an inverse.

$$MM^{-1} = I \qquad \text{The product of M and its inverse is identity.}$$

Let's apply this fact to our product and do some algebraic manipulation:

$$\begin{aligned}
(AB)(AB)^{-1} &= I && \text{By the definition of the inverse.} \\
A^{-1}(AB)(AB)^{-1} &= A^{-1}I && \text{Multiply both sides by } A^{-1}. \\
(A^{-1}A)B(AB)^{-1} &= A^{-1} && \text{Matrix multiplication is associative. } MI = M. \\
B(AB)^{-1} &= A^{-1} && \text{The product of inverses is I, so we drop } A^{-1}A. \\
B^{-1}B(AB)^{-1} &= B^{-1}A^{-1} && \text{Multiply both sides by } B^{-1}. \\
(AB)^{-1} &= B^{-1}A^{-1} && \text{The product of inverses is I, so we drop } B^{-1}B.
\end{aligned}$$