We want to undo a translation T. We need its inverse. What do we know about inverses? Well, we know the inverse's definition:

$$MM^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$$

The product of M and its inverse is identity.

What else do we know?

$$T = \begin{bmatrix} 1 & 0 & 0 & t_0 \\ 0 & 1 & 0 & t_1 \\ 0 & 0 & 1 & t_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translations have this form.

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 & ?_0 \\ 0 & 1 & 0 & ?_1 \\ 0 & 0 & 1 & ?_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

It's inverse has this form too; it's a translation.

$$TT^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The product of T and its inverse is identity.

Let's expand this last truth out a bit:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_0 \\ 0 & 1 & 0 & t_1 \\ 0 & 0 & 1 & t_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & ?_0 \\ 0 & 1 & 0 & ?_1 \\ 0 & 0 & 1 & ?_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Applying the mechanics of matrix multiplication, in which element (i, j) is the dot product of row i of the first operand with column j of the second, we can say:

$$\begin{aligned} 0 &= \begin{bmatrix} 1 & 0 & 0 & t_0 \end{bmatrix} \cdot \begin{bmatrix} ?_0 & ?_1 & ?_2 & 1 \end{bmatrix} && \text{Calculating element (1, 4).} \\ &= 1 \cdot ?_0 + 0 \cdot ?_1 + 0 \cdot ?_2 + t_0 \cdot 1 && \text{Let's expand that out.} \\ &= ?_0 + t_0 && \text{And simplify. 0s and 1s go away.} \\ ?_0 &= -t_0 && \text{We solve for } ?_0. \end{aligned}$$

We arrive at $?_1 = -t_1$ and $?_2 = -t_2$ in similar fashion. We plug these values back in to get T^{-1}:

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 & -t_0 \\ 0 & 1 & 0 & -t_1 \\ 0 & 0 & 1 & -t_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$